Note: Sections 7.1-7.4 – Angular velocity, Angular Acceleration, and how it compares to its linear counterparts will be covered in Chapter 8

7.5 - **Centripetal Acceleration** – the acceleration an object experiences when it moves at constant speed \( v \) in a circular path of radius \( r \).

Hmm... How does acceleration exist if an object moves at constant speed? Consider these two scenarios:

a) Is a car accelerating if it is moving at a constant speed along a straight road? Yes because the speed AND direction are constant, meaning constant \( v \), thus:

\[
a = \frac{\Delta v}{\Delta t} = \frac{V_f - V_i}{\Delta t} = \frac{0\text{ m/s}}{\Delta t} = 0\text{ m/s}^2
\]

b) Is a car accelerating if it is moving at a constant speed along a circular path with radius \( r \)? Yes – because it is constantly changing direction. This means the velocity is changing.

Don’t think so: put a cup of coffee on the dashboard as an accelerometer and perform this. But be sure to be turning toward your left while your passenger in the front seat video tapes the cup of coffee.
Derivation of the equation for centripetal acceleration:

1) triangle \( v-v-\Delta v \) and triangle \( r-r-\Delta s \) are similar triangles, so:

\[
\frac{\text{base}}{\text{side}} = \frac{\Delta v}{v} = \frac{\Delta s}{r}
\]

2) Solving for \( \Delta v \) and subbing into the acceleration equation

\[
\Delta v = \frac{v\Delta s}{r} \quad a = \frac{\Delta v}{\Delta t} = \frac{v\Delta s}{r\Delta t}
\]

3) and finally since

\[
v = \frac{\Delta s}{\Delta t} \quad a = \frac{v^2}{r}
\]

\( a_c \) is the radial acceleration radially (along the radius of the circle towards the center).

Remember: CENTRIPetal means “towards to CENTER!!”
Note: not on A.P. Physics B, but consider this:
Can an object moving in a circle have a tangential acceleration?

\[ a_t \text{ is the tangential acceleration.} \]

Hence, the total acceleration is the vector sum of \( a_t \) and \( a_c \).

\[ a = \sqrt{a_t^2 + a_c^2} \]

If the \( a_t \) is 0, the net acceleration is \( a_c \) as is the case when a car is moving at constant speed around a circle.

#1) A stunt driver at a carnival in India braves the well of death, or “Maut ka Kuua”. He drives around a circular path at a constant speed of 10 m/s. The radius of the circle is 5.5 m.

a) What is the magnitude of the centripetal acceleration? Direction?

(18.18 m/s\(^2\), toward the center)

b) If the time it takes to make 1 revolution doubles, the acceleration would be ______. (4.5 m/s\(^2\))
7.6 Centripetal Force – The force required to keep an object of mass $m$ moving at a speed $v$ in a circular path of radius $r$
For linear motion:
\[ \Sigma F = F_{\text{net}} = ma \]
It follows that for circular motion (when $a_t=0$):
\[ F_c = ma_c \quad F_c = \frac{mv^2}{r} \]
BIG NOTE: The CENTRIPETAL FORCE is the NET FORCE toward the center of the path acting on an object moving in a circular path of constant radius at constant speed.

#2) An object moves around a circle. Draw the $a_c$, velocity, and $F_c$ vectors at pts A, B, C, D.
A) Horizontal Circles – no tangential acceleration, only $a_c$

Demo: Turntable and Penny

#3) A coin sits on a turntable that is rotating at a constant speed as shown. You are looking at the turntable from above (a birds-eye view). In the position shown, what is the direction of the friction at this point?

Centripetal force and friction on a horizontal surface: if the only force holding an object in a circle is friction, then

$$\Sigma F_x = F_f \quad \text{or} \quad ma_c = \mu mg \quad \text{or} \quad \frac{v^2}{r} = \mu g$$

This is why a car making a sharp turn MUST SLOW DOWN, no matter what! The ratio of $v^2:r$ must satisfy the product of $\mu g$. If $r$ is too small such that $v^2/r > \mu g$, then the car______________
#4) Physics Olympics: A phonograph turntable is spinning at a certain velocity. A penny is to be placed on the turntable at a certain radius such that it does not slide off. Calculate the coefficient of Static friction between the penny and the turntable surface.

(\text{Group lab: calculate the max. static friction between turntable and penny—measure radius such that coin does not fly off})

Interesting. Here the penny slipped at a large radius, why?

Any point on a line directed radially out from center is going the same ANGULAR speed (all make one $360^\circ$ revolution in the same time $t$), but objects at a greater radius are traveling a greater circumference and thus a greater speed such that:

$$a = \frac{v^2}{r} = \left(\frac{2\pi r}{t}\right)^2 = \frac{4\pi^2 r^2}{t^2} = \frac{4\pi^2 r}{t^2}$$

This is why riding on the outer horses on a merry-go-round is the most fun. You get the greatest $F_c$. It’s also why the ride operator at the center of Starship 2000 at Rye Playland experiences near zero $a_c$. 
B) **Vertical Circles**—Time to introduce a tangential acceleration (g)

#5) A 50-kg child is on the Spin Meister, a ride at Great Adventure through 2006, spinning in a vertical circle with radius 6.8 m. The car’s hinges snap at point A, sending the child 13.4 m above the ground.

---

**a)** Find the speed of the child at the instant the hinges snap at pt A.  
**b)** Find the centripetal acceleration of the child at pt. A  
**c)** Find the centripetal force on the child at pt. A  
**d)** If the hinge snapped at point B, where would the child land?  
(a) 11.5 m/s (b) 19.4 m/s^2 (c) 971 N (b) 19 m)
#6) A 2 kg ball is swung in a vertical circle of radius 3 m. If the speed of the ball at the top is 6 m/s:

a) Find the tension in the cord at point B (at top) (4 N)

b) Find the speed of the ball at point D (at bottom) (12.5 m/s)

c) Find the tension in the cord at point D (at bottom) (124 N)

d) Find the tension in the cord at point A (\(v_A = 11.5 \text{ m/s, } T_A = 971 \text{ N}\))
Note: at point D, the $a_t = 0$, hence there is only $a_c$. But what about points A and C?

**At Point A:**

Note: Isn’t the total a in the same direction as the net force when you do a free body diagram on the ball at point A?

Circular motion is peculiar. One net force is responsible for the CHANGE IN DIRECTION, and the other net force is responsible for the CHANGE IN MAGNITUDE. So, just as we break up the x and y components of velocity when calculating the motion of projectiles, we break up the radial and tangential components of the acceleration. The net force for the circular component is still directed toward the center, and the net force for the tangential acceleration is directed tangentially and is often called a RESTORING FORCE.
DEMO: Swing 1 kg mass as shown (attach spring scale to the line)

#7) An 1-kg mass drops from rest at point A. Find the tension in the cord at points A, B and C. (A and C) T=0 N (B) T= 30 N
A 60-kg woman on massless steel cable of length 6.5 m (measured from pivot point to center of woman’s mass) starts from rest at 60° from the vertical on the Tarzan Swing at Mountain Creek Water Park. Being afraid, this young woman holds on to the cable and swings back and forth between point C and A several times before letting go and falling into the water.

A) What is the tension in the cord at the bottom (pt. B)? (1200 N, twice her weight!)

B) What is the tension in the cord at the maximum height? (pt. C or pt. A)? (300 N, half her weight)

C) What is the restoring force at the highest point in her swing? (520 N)

D) What is the $a_t$ at point A? (8.66 m/s²)

E) What is the $a_t$ at point B? (Hence, what is the restoring force at point B? (0 m/s², 0 N))
#9) A block of mass $M$ is dropped from rest. The length of the cord is $L$.

a) Find the tension in the cord at the bottom in terms of $M$, $g$, and $L$. ($T = 3Mg$)

b) What is the net force at the bottom? ($\Sigma F = 2Mg$)

Now go home and do HW # 29,33,40 and study example 7.6 on p 182
C) **Vertical and Horizontal circles COMBINED** – the Swing Ride Problem.

#10) A ball is twirled in a horizontal circle at constant speed \( V \).

\[ \Theta \]

a) What is the greatest angle \( \Theta \) possible?
b) Draw an F.B.D. on the ball.
c) What is the net Force acting on the ball?
D) The Loop de Loop problem - moving in a circle with a normal force only (no tension)

#11) A cart of mass $m$ slides down a frictionless slope and then enters a “loop de loop” with radius $R$. What is the cart’s minimum height above the ground that it needs to complete the loop de loop? Answer in terms of $R$ only. ($h=5R/2$)
7.7 “Centrifugal Forces” - What are they, and do they really exist?
A person is sitting in the passenger seat of a car with their iPhone on the dashboard and their window open all the way (note: iPhone does not have that nice rubberized protective cover). What happens to the iPhone sitting on the dashboard if you make a turn around the bend as shown?

Some would say it’s a centrifugal force pushing the iPhone out the window to the right as the car turns left. This is because the car is a NON-inertial (or accelerated) reference frame, and Newton’s first law of motion is only true for INERTIAL (constant velocity) reference frames.

What is the real reason the phone is “thrown” out the window?
Prior to turning, the phone and the car are moving together at constant velocity. As the car makes the left turn around the bend, the car’s tires provide the necessary centripetal force to move the car in a circular path. However, the friction under the phone is NOT enough to keep the phone in the circular path, so it continues in a straight line while the car turns.

Remember though, as a human being on Earth, you are moving in a non-inertial reference frame, and it is easier to study the motion of objects in such a frame by introducing fictitious forces, such as the Coriolis Force that accounts for the rotation of air pressure systems in meteorology, and the periodic motion of the Foucault Pendulum.
7.8 **Newton’s Law of Gravitation**

Every object in the universe attracts every other object with a force that is directly proportional to the product of their masses and inversely proportional to the square of the distance between them.

**Formula:** \[ F_g = G \frac{m_1 m_2}{r^2} \]

\[ G = 6.673 \times 10^{-11} \text{ Nm}^2 \text{kg}^{-2} \]

Quick History: Did Newton determine G? No. He determined the inverse-square relationship to be the same for objects acted on by gravity near earth’s surface (like a falling apple) and the moon moving around the earth at a great distance. But how did they figure out the distance between earth and moon, earth and sun, etc. Trigonometry!

For more info see

### #12) Find the gravitational force that the Sun exerts on the Earth. \((3.55 \times 10^{22} \text{N})\)

- Mass of Earth = \(5.98 \times 10^{24} \text{ kg}\)
- Mass of Sun = \(1.991 \times 10^{30} \text{ kg}\)
- Sun – Earth distance = \(1.496 \times 10^{11} \text{ m}\)
#13) Two tennis balls are placed on a desk, each of mass 60 g and separated by 1.3 m.
A) Find the gravitational force between them. $1.42 \times 10^{-13} \text{N}$
B) Do they accelerate towards each other? (assume $\mu_s = .2$) (No)

#14) Find the net gravitational force on $m_3$ due to masses $m_1$ and $m_2$.
Each mass = $m$ ($\frac{89Gm^2}{1600}$)}
#15) Find the gravitational force on \( m_1 \) due to masses \( m_2 \) and \( m_3 \).

\( (6.71 \times 10^{19} \text{N}) \)

\[ m_1 = 1.3 \times 10^{22} \text{ kg} \]
\[ m_2 = 3.6 \times 10^{22} \text{ kg} \]
\[ m_3 = 1.4 \times 10^{22} \text{ kg} \]
\[ d_{12} = 2.8 \times 10^7 \text{ m} \]
\[ d_{13} = 1.5 \times 10^7 \text{ m} \]

Now go home and do HW #42,43,44
A) **Law of Gravitation and Weight**

#16) A 70-kg kangaroo is sitting on the Earth, about to fight his trainer. Calculate the gravitational force of attraction between the kangaroo and the Earth. (The radius of the Earth is $6.37 \times 10^6$ m) (688N)

Hence, the gravitational Force is also the object’s ____________.

From $F_{grav} = W$, derive an expression for the acceleration due to gravity $g$. 
#17) In example 16, find the kangaroo’s weight if it is:
A) fighting 1 Earth Radius above the surface. \(W/4\)
B) fighting 2 Earth Radii above the surface. \(W/9\)
C) fighting 3 Earth Radii above the surface. \(W/16\)

B) **Centripetal Force and Gravitational Force Relationship**

#18) Assume that the Earth orbits the Sun in a circular path (actually elliptical, but close to a circle). Calculate the centripetal force on the Earth about the Sun. (using leap year of 365.25 d, \(F_c = 3.546 \times 10^{22} \text{N}\))

- Mass of Earth = 5.98 \(\times 10^{24}\) kg
- Mass of Sun = 1.991 \(\times 10^{30}\) kg
- Sun – Earth distance = 1.496 \(\times 10^{11}\) m

So, based on your calculations on #18 and on #12 on p. 16, what is the relationship between \(F_g\) and \(F_c\)?

**For an object orbiting another object,** \(F_g = F_c\)

From \(F_g\) and \(F_c\) derive the formula for the orbital velocity of the Earth.

**Note:** the orbital velocity is INDEPENDENT of _____________

Now go home and do HW #47
Question: As an object moves higher above the ground, the PE\(_g\) ___________. But...

As you continue to move this object higher and higher away from the Earth (leaving the Earth’s atmosphere), the PE\(_g\) ___________. At Infinity, PE\(_g\) = ZERO!!

Near the Earth’s surface PE\(_g\) = mgh is valid.

At very large distances from the Earth’s surface we use:

\[
U = -\frac{GM_e m}{r}
\]

Where:
U = Potential Energy
r = distance from the Earth’s center
U is negative because the reference level is chosen as:
U=0 at r=\(\infty\)
#17) A 1-kg satellite is $2 \times 10^6$ m above the Earth’s surface. Calculate the Potential Energy at this location:

(a) relative to infinity. $(-4.77 \times 10^7 \text{ J})$

$\text{M}_e = 5.98 \times 10^{24} \text{ kg}$
$\text{r}_e = 6.37 \times 10^6 \text{ m}$

(b) relative to the earth’s surface. $(1.49 \times 10^7 \text{ J or } 2 \times 10^7 \text{ J using } \text{mgh})$

$\text{M}_e = 5.98 \times 10^{24} \text{ kg}$
$\text{r}_e = 6.37 \times 10^6 \text{ m}$
8.1 **What is Torque?**

**DEMO: Student push door at hinge, Torque feeler**

**Torque** – The ability of a force to rotate a body about a fixed point.

Mathematically, Torque is the cross product of the force applied and perpendicular distance to the fixed point.

\[ \tau = F \perp \times d = F d \sin \theta \]

**UNITS of Torque:**___________
#1) In each, calculate the net torque

Note: the textbook defines a CCW torque as +, CW torque as -

A) $20 \text{ N} \cdot 3 \text{ m} = 60 \text{ Nm}$

B) $20 \text{ N} \cdot 3 \text{ m} = 60 \text{ Nm}$

C) $30 \text{ N} \cdot 2 \text{ m} = 60 \text{ Nm}$

D) $30 \text{ N} \cdot 2 \text{ m} = 60 \text{ Nm}$

$10 \text{ N} \cdot 2 \text{ m} = 20 \text{ Nm}$

Total: $60 \text{ Nm} + 60 \text{ Nm} + 60 \text{ Nm} + 20 \text{ Nm} = 200 \text{ Nm}$
Now go home and do HW# 1, 2, 4, 5

#2) A fisherman catches a fish as shown. If the fish exerts a force of 100 N, find the torque about the handle of the 2 m fishing rod. (195 Nm)
8.2 Torque and the 2\textsuperscript{nd} condition for Equilibrium

Question: From chapter 4 we said that an object is in equilibrium if: \( \Sigma F_x = 0 \) and \( \Sigma F_y = 0 \)

If two people are pushing on a log as shown, is the system in equilibrium? (Are \( \Sigma F_x = 0 \) and \( \Sigma F_y = 0 \) ?)

The SECOND CONDITION FOR EQUILIBRIUM:
A system is in equilibrium if:
1) \( \Sigma F_x = 0 \) and \( \Sigma F_y = 0 \)
AND:
2) \( \Sigma \tau = 0 \)
#3) Are the following in Equilibrium?

A) ![Diagram of forces](image1)

Yes!

B) ![Diagram of forces](image2)

No!

This one explains why Scarlet took a tumble.

#4) A window washer is standing on a uniform scaffold supported by vertical ropes at each end. The scaffold weighs 200 N and is 3 m long. What is the tension in each rope when a 700 N worker stands 1 m from one end?
#5) Newman (100 kg) and Jerry (70 kg) sit on opposite sides of a seesaw 3 m long. The uniform board weighs 200 N. Where must the fulcrum be placed so that the system is in equilibrium? (0.24 m off center, closer to Newman)

Now go home and do HW #7, 8, 12
7.1 What is angular velocity?

Linear velocity is defined as \[ v = \frac{\Delta d}{\Delta t} \] (position is changing per unit time)

Angular velocity is defined as \[ \omega = \frac{\Delta \Theta}{\Delta t} \text{ (IN RADIANS)} \]
(Angular displacement is changing per unit time)
Units of Angular velocity:

Why use Radians?
When \( \Theta \) is measured in radians:
\[ \Theta = \frac{s}{r} \]

Thus: when the point moves through 1 revolution:
\[ \Theta = \frac{s}{r} = \frac{2\pi r}{r} = 2\pi \text{ radians or 6.28 radians} \]
Thus 6.28 rad = 360°
Note: The degree is not based on a ratio like a radian. It’s based on the fact that the “circular year” takes 360 days (based on the original ancient Babylonian estimate)

#6) A record player makes $33\frac{1}{3}$ revolutions per minute.
   a) Find the angular velocity. \(3.49 \text{ rad/s}\)
   b) Find the frequency in Hz. \(0.555 \text{ Hz}\)
   c) What is the relationship between an object’s angular velocity and its frequency?
7.4 Relationship between linear and angular velocity

#7) Point P moves about the record player at a constant rate and makes one revolution in 1.33s.
A) Find the angular velocity. (4.72 rad/s)

B) Find its linear velocity (0.5664 m/s)

Point Q moves about the record player at a constant rate and makes one revolution in 1.33s.
A) Find the angular velocity __________ (4.72 rad/s)
B) Find its linear velocity __________ (0.2832 m/s)

** What is the relationship between the linear and angular velocity? 
\[ v = r \omega \]

***Points at different radial distances on a rotating rigid body have different linear speeds along their circular paths, however, every point on a rotating body has the SAME angular velocity.
7.2 Angular Acceleration-

When an object’s angular velocity is changing, we say that it has angular Acceleration $\alpha$.

$\omega = \text{angular velocity}$  
$\alpha = \text{angular acceleration}$  
$\Delta\theta = \text{change in angle}$

And now...

Your favorite kinematics equations: In ANGULAR FORM:

El Grande:  
$$\Delta\theta = \omega_i t + \frac{1}{2} \alpha t^2$$

FIAT:  
$$\omega_f = \omega_i + \alpha t$$

The forgotten one:  
$$\omega_f^2 = \omega_i^2 + 2\alpha \Delta\theta$$

Hand Out – Angular Problem Practice
Harder Rotational Problems:
#1) In a popular amusement park ride (The rotor), a cylinder of radius 3m is set in rotation at an angular speed of 5 rad/s. The floor then drops away, leaving the riders suspended against the wall in a vertical position. What minimum coefficient of friction between a rider’s clothing and the wall of the cylinder is needed to keep the rider from slipping? (0.133)
A copper block rests 45cm from the center of a steel turntable. The coefficient of static friction between the block and the surface is 0.44. The turntable starts from rest and rotates with a constant angular acceleration of 0.4 rad/s². After what time interval will the block start to slip on the turntable? (Hint: first find ω_f, then find t.) (7.8 sec)
Conservation of Angular Momentum
(only tested on qualitative, not quantitative understanding)

In order to understand angular momentum, one needs to understand the concept of **Moment of Inertia**.
When a net torque causes an object to accelerate angularly, the RATIO of the net torque $\tau$ to the angular acceleration $\alpha$ is called the moment of **Inertia** $I$.  

$$\Sigma \tau = I \alpha$$

Every rigid body has a different formula to calculate its moment of inertia. But if you study the examples below, you will see that:

Objects with mass distributed farther from the axis of rotation have higher moments of inertia. An object with a large $I$ will need a larger $\tau$ to rotate it at a certain $\alpha$.

Note: Moment of Inertia is the rotational counterpart to _______.

**Examples:**

- **Solid cylinder or disc, symmetry axis**
  $$I = \frac{1}{2} MR^2$$

- **Hoop about symmetry axis**
  $$I = MR^2$$

- **Solid sphere**
  $$I = \frac{2}{5} MR^2$$

- **Rod about center**
  $$I = \frac{1}{12} ML^2$$

- **Solid cylinder, central diameter**
  $$I = \frac{1}{4} MR^2 + \frac{1}{12} ML^2$$

- **Hoop about diameter**
  $$I = \frac{1}{2} MR^2$$

- **Thin spherical shell**
  $$I = \frac{2}{3} MR^2$$

- **Rod about end**
  $$I = \frac{1}{3} ML^2$$
#8) In the example of a rotating baton we calculate the moment of inertia as $I = \Sigma mr^2$ (assuming that the $m_{\text{baton}} \ll m_{\text{ends}}$, or two point masses at distance $r$ from center of rotation)

A) Calculate the moment of Inertia for the system. $(4.5 \text{ kgm}^2)$

B) How could you increase the system’s moment of inertia?
And now... Conservation of Angular Momentum

Angular Momentum is the product of the object’s moment of inertia and its angular velocity.

Thus, \( I_i \omega_i = I_f \omega_f \)

#9) Explain how an Olympic figure skater can increase and decrease his/her angular velocity. (Sketch the skater)

**DEMO: Spinning Chair**
**Youtube:** Merry Go Round of Death
**DEMO: Spinning Bicycle Wheel**
**Youtube** Conservation of Angular Momentum vid (good explanation for spinning wheel)

Youtube: Cat Falling from tree plus explanatory videos
Angular Momentum of a point particle

Since there is a relationship between the linear velocity and angular velocity, the angular momentum $L$ of a point particle can also be calculated by:

$$L = I\omega = (mr^2)(v/r)$$

Thus,

$L = mv \cdot r$

Angular momentum = Linear momentum x radius
#10)
As the Earth approaches the sun, which remains constant?
   A)  Speed
   B)  Kinetic Energy
   C)  Potential Energy
   D)  Angular Momentum
   E)  Centripetal Acceleration

Chapter 7-8: Done.